JOURNAL OF COMPUTATIONAL PHYSICS 6, 322–325 (1970)

A Simplified MAC Technique for Incompressible Fluid Flow Calculations

An alternative formulation has been found for the MAC method, which simplifies considerably its application to the numerical solution of time-dependent, incompressible fluid flow problems. The new version requires fewer boundary conditions and eliminates the boundary inhomogeneities of the Poisson equation, allowing for more efficient solution techniques. This simplified-MAC (SMAC) method has been described in detail in a report that contains the full equations, the flow diagram and Fortran listing, and sample calculations.

The Marker-and-Cell (MAC) method [1–10] was developed for the time-dependent, numerical solution of confined or free-surface flow of a viscous, incompressible fluid. Despite the cumbersome nature of the boundary conditions, for which precise consistency requires tedious derivations in each new case, the method has found applicability to a wide variety of problems.

The purpose of this note is to describe briefly a simplified-MAC (SMAC) [11] technique that eliminates most of the boundary condition difficulties, and also extends the scope of investigations to which the calculations can be applied. It utilizes a three-phase procedure that resembles the technique used by Chorin [12] for confined fluid flows.

A detailed description of the SMAC technique is presented in [11], which contains all of the equations and boundary conditions. Also included in the report is a complete computer program that can be used for problems involving the following features:

1. Confined flow or free-surface flow,
2. Plane or cylindrical coordinates,
3. An input wall,
4. A rigid obstacle with free-slip or no-slip walls,
5. An output wall which may be continuative, or have a prescribed suction velocity.

The program is described by a step-by-step discussion, by complete flow sheets, by a Fortran listing, and by the results of some sample calculations.

SMAC-method calculations utilize an Eulerian finite-difference mesh of rectangular cells, together with a time advancement through finite intervals, or cycles. Each cycle contains three basic phases, as follows:

1. The full Navier–Stokes equations are modified by substitution of a pseudopressure field in place of the true pressures. The pseudo pressures are arbitrarily specified except in free-surface cells; for example, the value in each full cell may be zero, but in any case the choice is based on matters of calculational efficiency and does not require solution of a Poisson equation. In each free-surface cell, the pseudopressure is equal to the true pressure that is necessary to satisfy the free-surface normal stress condition. Otherwise, only the physical boundary conditions on the velocity field itself are required. A finite-difference approximation to the modified Navier–Stokes equations is used for the explicit calculation of a tentative new velocity field, which then contains the correct vorticity for each mesh intersection point, but does not satisfy the incompressibility condition (vanishing divergence).

2. The tentative new velocity field is modified to its final values in such a way as to preserve the vorticity but bring the divergence to zero. This is accomplished by setting the difference between final and tentative velocities equal to the finite-difference gradient of a potential function. The potential function is then found by solving a finite-difference Poisson equation, for which the boundary conditions are strictly homogeneous. As a result, the Poisson equation can be solved more easily and efficiently than the corresponding Poisson equation for pressure in the MAC method. With this potential function, the final velocities can be found.

3. Massless marker particles imbedded in the fluid are moved, so that the configuration of surface cells can be sensed for the pseudopressure boundary conditions. These particles also serve to describe the changes of internal configuration of the fluid.

The basic MAC method also uses the same Eulerian mesh and marker particles, with cyclic advancement of the fluid configuration to obtain the solution, but differs appreciably in the solution technique. In MAC, the pressure for each cell is obtained by solving a Poisson equation, for which the source term is a function of the velocities, derived in such a manner that the resulting momentum equations satisfy the incompressibility condition. The full Navier–Stokes equations are then used to find new velocities throughout the mesh.

The advantage of SMAC is not that new problems can be solved, but that it is an order of magnitude simpler to manipulate when adapting it to various mesh boundary configurations. This results from having fewer boundary conditions to complicate the programming logic. Also, the Poisson equation for the potential function eliminates boundary inhomogeneities that appear in the MAC Poisson pressure equation. Thus, it lends itself more readily to direct solution techniques, which have been known to cut the solution time of such equations by appreciable amounts, in comparison with iterative solution techniques.

The calculation examples used to illustrate [11] are the following:

1. The motion of water behind a broken dam, with both no-slip and free slip walls,
2. The sloshing of fluid in a tank,
3. The splash of a drop of one fluid falling into another,
4. The flow of fluid over a step,
5. The waterfall problem,
6. The problem of controlled drainage.

Comparisons of results have been made with those of other computing methods, and of laboratory experiments, and agreement is excellent.

REFERENCES

1. F. H. HARLOW, J. P. SHANNON, AND J. E. WELCH, Liquid waves by computer, *Science* **149** (1965), 1092.
2. F. H. HARLOW, J. E. WELCH, J. P. SHANNON, AND B. J. DALY, The MAC Method, Los Alamos Scientific Lab. Rep. No. LA-3425 (1965).
3. F. H. HARLOW AND J. E. WELCH, Numerical calculation of time-dependent viscous incompressible flow, *Phys. Fluids* **8** (1965), 2182.
4. F. H. HARLOW AND J. E. WELCH, Numerical study of large amplitude free surface motions, *Phys. Fluids* **9** (1965), 842.
5. B. J. DALY, A numerical study of two-fluid Rayleigh–Taylor instability, *Phys. Fluids* **10** (1967), 297.
6. F. H. HARLOW AND J. P. SHANNON, The splash of a liquid drop, *J. Appl. Phys.* **38** (1967), 3855.
7. F. H. HARLOW AND J. P. SHANNON, Distortion of a splashing liquid drop, *Science* **157**, No. 3788 (1967), 547.
8. B. J. DALY AND W. E. PRACHT, A numerical study of density current surges, *Phys. Fluids* **11** (1968), 15.
9. C. W. HIRT AND J. P. SHANNON, Free surface stress conditions for incompressible flow calculations. *J. Comput. Phys.* **2** (1968), 403.
10. B. J. DALY, A technique for including surface tension effects in hydrodynamic calculations, *J. Comput. Phys.* **4** (1969), 97.
11. A. A. AMSDEN AND F. H. HARLOW, The SMAC method, Los Alamos Scientific Lab. Rep. No. LA-4370 (1970).

12. A. J. CHORIN, Numerical solution of the Navier–Stokes equations, *Math. Comput.* **22** (1968), 745.

RECEIVED: February 9, 1970

REVISED: March 17, 1970

ANTHONY A. AMSDEN AND FRANCIS H. HARLOW*

University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87544

* This work was performed under the auspices of the United States Atomic Energy Commission.